$$\mathcal{L}\{u(t-a)f(t)\} = e^{-as}\mathcal{L}\{f(t+a)\}$$

$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = u(t-a)\underbrace{\mathcal{L}^{-1}\{F\}(t-a)}_{\substack{\text{INVERSE LAPLACE OF } F \\ \text{WITH } t \text{ REPLACED BY } t-a}}$$

$$\mathcal{L}^{-1}\left\{e^{-\pi s}\left(\frac{s+2}{s^2+1}\right)\right\} = u(t-\pi)(-\cos t - 2\sin t)$$

$$\downarrow$$

$$\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+1} + \frac{2}{s^2+1}\right\}$$

$$= \cos t + 2\sin t$$

$$\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+1}\right\}(t-\pi) = \cos(t-\pi) + 2\sin(t-\pi)$$

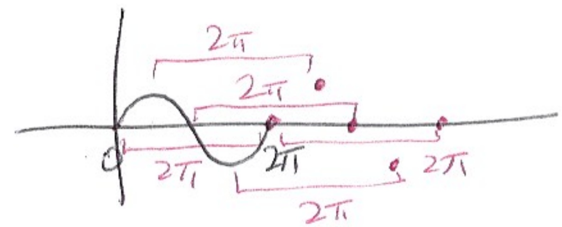
$$= -\cos t - 2\sin t$$

PERIODIC FUNCTIONS

A FUNCTION f IS PERIODIC WITH PERIOD T

IF $f(t+T) = f(t)$

eg. $\sin(x+2\pi) = \sin x$ SO $\sin x$ IS PERIODIC WITH PERIOD T



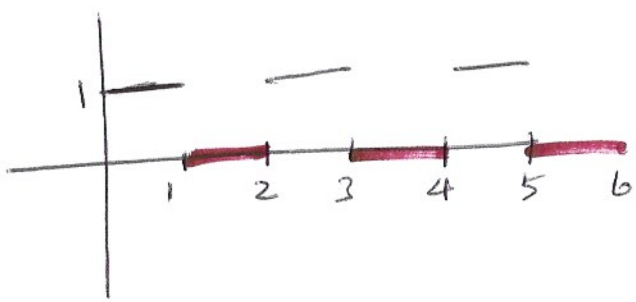
[WE ONLY CONSIDER $f : [0,\infty) \to \mathbb{R}$]

$$f_T = \begin{cases} 1, & 0<t<1 \\ 0, & 1<t<2 \\ 0, & t>2 \end{cases} = \begin{cases} 1, & 0<t<1 \\ 0, & t>1 \end{cases}$$

$T=2$

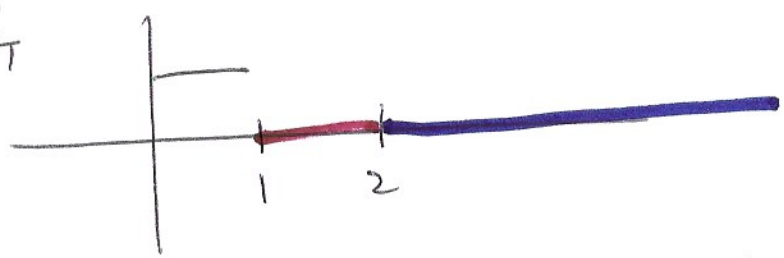
$$= 1 + u(t-1)(0-1)$$
$$= 1 - u(t-1)$$

f



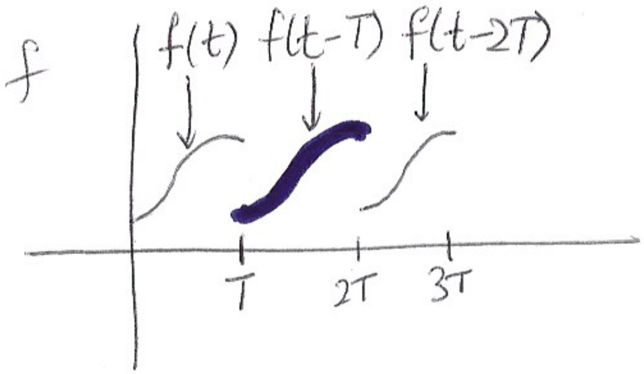
$$\mathcal{L}\{f_T\} = \mathcal{L}\{1 - u(t-1)\}$$
$$= \frac{1}{s} - e^{-1s}\frac{1}{s}$$
$$= \frac{1-e^{-s}}{s}$$

f_T



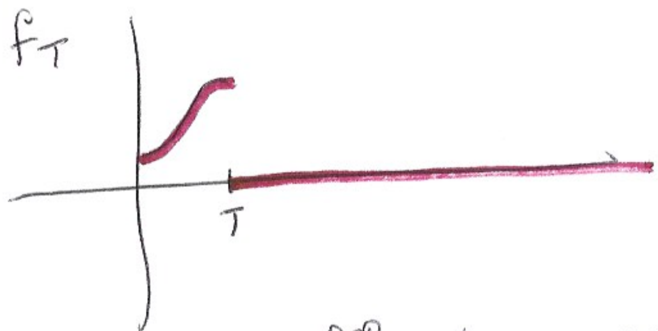
$$\mathcal{L}\{f\} = \frac{\frac{1-e^{-s}}{s}}{1-e^{-2s}} = \frac{\frac{\cancel{1-e^{-s}}}{s}}{(1+e^{-s})\cancel{(1-e^{-s})}} = \frac{1}{s(1+e^{-s})}$$

LET f BE A PERIODIC FUNCTION OF PERIOD T



$$f_T(t) = \begin{cases} f(t), & 0 < t < T \\ 0, & t > T \end{cases}$$

$$\mathcal{L}\{f_T\} = \int_0^\infty e^{-st} f_T(t)\,dt$$
$$= \int_0^T e^{-st} f(t)\,dt$$



$u = t - T$: $t = 2T \to u = T$; $t = T \to u = 0$

$t = u + T \to dt = du$

$u = t - 2T$: $t = 3T \to u = T$; $t = 2T \to u = 0$

$t = u + 2T \to dt = du$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$
$$= \underbrace{\int_0^T e^{-st} f(t)\,dt}_{} + \int_T^{2T} e^{-st} f(t-T)\,dt + \int_{2T}^{3T} e^{-st} f(t-2T)\,dt + \cdots$$
$$= \mathcal{L}\{f_T\} + \int_0^T e^{-s(u+T)} f(u)\,du + \int_0^T e^{-s(u+2T)} f(u)\,du$$
$$= \mathcal{L}\{f_T\} + e^{-sT}\int_0^T e^{-su} f(u)\,du + e^{-2sT}\int_0^T e^{-su} f(u)\,du$$
$$= \mathcal{L}\{f_T\} + e^{-sT}\mathcal{L}\{f_T\} + e^{-2sT}\mathcal{L}\{f_T\} + \cdots$$

$$= \mathcal{L}\{f_T\}\left(1 + \underset{*e^{-sT}}{e^{-sT}} + \underset{*e^{-sT}}{e^{-2sT}} + \underset{*e^{-sT}}{e^{-3sT}} + \underset{*e^{-sT}}{e^{-4sT}} + \dots\right)$$

$$= \mathcal{L}\{f_T\} \cdot \frac{1}{1-e^{-sT}}$$

INFINITE GEOMETRIC SERIES

$$S = \frac{a_1}{1-r}$$

$$\mathcal{L}\{f\} = \frac{\mathcal{L}\{f_T\}}{1-e^{-sT}}$$

FIND $\mathcal{L}^{-1}\left\{\frac{1}{s(1+e^{-s})}\right\}$

☆ CAN'T USE PARTIAL FRACTIONS UNLESS BOTH NUMERATOR & DENOMINATOR ARE POLYNOMIALS

RECALL $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 + (-x) + (-x)^2 + (-x)^3 + (-x)^4 + \dots$$

$$= 1 - x + x^2 - x^3 + x^4 - \dots$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s} \cdot \frac{1}{1+e^{-s}}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s}\left(1 - e^{-s} + e^{-2s} - e^{-3s} + e^{-4s} - \dots\right)\right\}$$

$$= 1 - u(t-1) + u(t-2) - u(t-3) + u(t-4) \dots$$

$$= \begin{cases} 1, & \text{IF } t<1 \\ 0, & \text{IF } 1<t<2 \\ 1, & \text{IF } 2<t<3 \\ 0, & \text{IF } 3<t<4 \end{cases}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\cdot e^{-ks}\right\} = u(t-k)\cdot 1$$

$$\downarrow$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}(t-k) = 1$$